PRECALCULUS REVIEW FINAL EXAM NAME _____ MOD ____

CHAPTERS 6-10

Find the exact value of the expression.

1) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ 1) _____

A) $\frac{-\pi}{4}$ B) $\frac{-3\pi}{4}$ C) $\frac{3\pi}{4}$ D) $\frac{\pi}{4}$

2) $\cos\left(\sin^{-1}\frac{1}{4}\right)$ 2) _____

A) $\frac{\sqrt{15}}{2}$ B) $\frac{4\sqrt{15}}{15}$ C) $\frac{2\sqrt{15}}{15}$ D) $\frac{\sqrt{15}}{4}$

3) $\sec\left(\tan^{-1}\frac{\sqrt{3}}{3}\right)$ 3) _____

A) $\frac{2\sqrt{3}}{3}$ B) $\frac{1}{2}$ C) $\sqrt{3}$ D) 2

4) $\sin 215^\circ \cos 95^\circ - \cos 215^\circ \sin 95^\circ$ 4) _____

A) $-\frac{\sqrt{3}}{2}$ B) $-\frac{43}{12}$ C) $\frac{\sqrt{3}}{2}$ D) $-\frac{1}{2}$

5) $\cos\left(\frac{2\pi}{9}\right)\cos\left(\frac{\pi}{18}\right)+\sin\left(\frac{2\pi}{9}\right)\sin\left(\frac{\pi}{18}\right)$ 5) _____

A) $\frac{1}{4}$ B) $\frac{\sqrt{3}}{2}$ C) 1 D) $\frac{1}{2}$

Use the information given about the angle θ, $0 \le \theta \le 2\pi$, to find the exact value of the indicated trigonometric function.

6) $\sin\theta = \frac{1}{4}$, $0 < \theta < \frac{\pi}{2}$ Find $\sin\frac{\theta}{2}$. 6) _____

A) $\frac{\sqrt{10}}{4}$ B) $\frac{\sqrt{8+2\sqrt{15}}}{4}$ C) $\frac{\sqrt{8-2\sqrt{15}}}{4}$ D) $\frac{\sqrt{6}}{4}$

7) $\csc\theta = \frac{13}{12}$, $\frac{\pi}{2} < \theta < \pi$ Find $\cos(2\theta)$. 7) _____

A) $-\frac{119}{169}$ B) $-\frac{120}{169}$ C) $\frac{119}{169}$ D) $\frac{120}{169}$

8) $\sin\theta = \frac{4\sqrt{3}}{7}$, $\tan\theta < 0$ Find $\sin(2\theta)$. 8) _____

A) $\frac{-8\sqrt{3}}{49}$ B) $\frac{8\sqrt{3}}{49}$ C) $\frac{47}{49}$ D) $-\frac{47}{49}$

Simplify the trigonometric expression by following the indicated direction.

9) Rewrite in terms of sine and cosine: $\tan\theta \cdot \cot\theta$ 9) _____

10) Multiply $\frac{\cos\theta}{1-\sin\theta}$ by $\frac{1+\sin\theta}{1+\sin\theta}$ 10) _____

11) Rewrite over a common denominator: $\frac{1}{1-\cos\theta}+\frac{1}{1+\cos\theta}$ 11) _____

12) Multiply and simplify: $\frac{(\cot\theta+1)(\cot\theta+1)-\csc^2\theta}{\cot\theta}$ 12) _____

13) Factor and simplify: $\frac{6\cos^2\theta+7\cos\theta+1}{\cos^2\theta-1}$ 13) _____

Prove the identity

14) $\frac{\csc\theta\cot\theta}{\sec\theta}=\cot^2\theta$ 14) _____

15) $\tan\theta(\cot\theta-\cos\theta)=1-\sin\theta$ 15) _____

16) $\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$ 16) _____

Solve the equation on the interval $0\le\theta<2\pi$.

17) $\cos^2\theta+2\cos\theta+1=0$ 17) _____

18) $\sin^2\theta+\sin\theta=0$ 18) _____

19) $\sin(2\theta)+\sin\theta=0$ 19) _____

Use a graphing utility to solve the equation on the interval $0^\circ\le x<360^\circ$. Express the solution(s) rounded to one decimal place.

20) $\tan^2 x+5\tan x+3=0$ 20) _____

Find the area of the triangle. If necessary, round the answer to two decimal places.

21) $a=4, b=5, c=7$ 21) _____

A) 3.46 B) 10.01 C) 16.01 D) 9.80

22) $\alpha=40^\circ, b=11, c=4$ 22) _____

A) 12.14 B) 14.14 C) 16.85 D) 18.85

Two sides of a right triangle ABC (C is the right angle) are given. Find the indicated trigonometric function of the given angle. Give exact answers with rational denominators.

23) Find sec B when $a=7$ and $b=4$. 23) _____

A) $\frac{4\sqrt{65}}{65}$ B) $\frac{7\sqrt{65}}{65}$ C) $\frac{\sqrt{65}}{7}$ D) $\frac{7\sqrt{65}}{4}$

Two sides and an angle are given. Determine whether the given information results in one triangle, two triangles, or no triangle at all. Solve any triangle(s) that results.

24) $a = 4, \quad b = 8, \quad \alpha = 75°$ 24) _____

A) one triangle
$\beta = 39°, \gamma = 66°, c = 16$

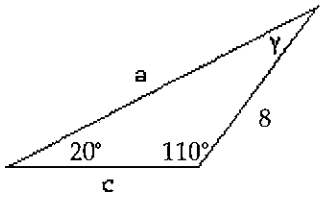
B) one triangle
$\beta = 37°, \gamma = -48°, c = 14$

C) one triangle
$\alpha = 38°, \gamma = 67°, c = 12$

D) no triangle

Solve the triangle.

25) 25) _____



26) $a = 6, b = 8, \gamma = 70°$ 26) _____

Solve the problem.

27) A surveyor standing 66 meters from the base of a building measures the angle to the top of the building and finds it to be 38°. The surveyor then measures the angle to the top of the radio tower on the building and finds that it is 50°. How tall is the radio tower? 27) _____

28) Two surveyors 180 meters apart on the same side of a river measure their respective angles to a point between them on the other side of the river and obtain 54° and 68°. How far from the point (line-of-sight distance) is each surveyor? Round your answer to the nearest 0.1 meter. 28) _____

29) A famous golfer tees off on a long, straight 478 yard par 4 and slices his drive 18° to the right of the line from tee to the hole. If the drive went 274 yards, how many yards will the golfer's second shot have to be to reach the hole? 29) _____

The polar coordinates of a point are given. Find the rectangular coordinates of the point.

30) $(-3, -135°)$ 30) _____

A) $\left(\frac{-3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$ B) $\left(\frac{3\sqrt{2}}{2}, \frac{-3\sqrt{2}}{2}\right)$ C) $\left(\frac{-3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$ D) $\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$

31) $\left(-9, \frac{2\pi}{3}\right)$ 31) _____

A) $\left(\frac{9}{2}, \frac{9\sqrt{3}}{2}\right)$ B) $\left(\frac{9}{2}, \frac{-9\sqrt{3}}{2}\right)$ C) $\left(-\frac{9}{2}, \frac{9\sqrt{3}}{2}\right)$ D) $\left(-\frac{9}{2}, \frac{-9\sqrt{3}}{2}\right)$

32) $(4, 70°)$ Round the rectangular coordinates to two decimal places. 32) _____

A) (4.01, 1.59) B) (1.37, 3.76) C) (1.59, 4.01) D) (3.76, 1.37)

The rectangular coordinates of a point are given. Find polar coordinates for the point.

33) (0, −4) 33) _____

A) $\left(4, \frac{\pi}{2}\right)$ B) $(4, \pi)$ C) $(4, 0)$ D) $\left(4, -\frac{\pi}{2}\right)$

Write the complex number in polar form. Express the argument in degrees, rounded to the nearest tenth, if necessary.

34) −5i 34) _____

A) $5(\cos 270° + i \sin 270°)$ B) $5(\cos 180° + i \sin 180°)$

C) $5(\cos 0° + i \sin 0°)$ D) $5(\cos 90° + i \sin 90°)$

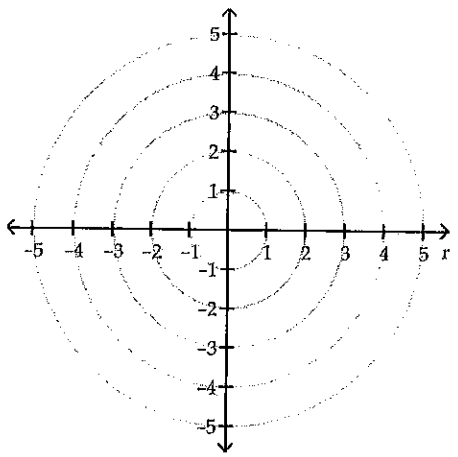
Solve the problem.

35) Plot the point $\left(4, \frac{\pi}{6}\right)$ and find other polar coordinates (r, θ) of the point for which: 35)

(a) $r > 0$, $-2\pi \le \theta < 0$

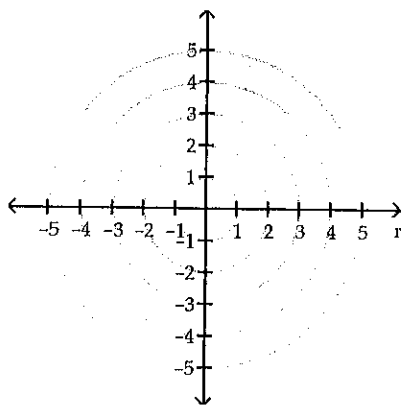
(b) $r < 0$, $0 \le \theta < 2\pi$

(c) $r > 0$ $2\pi \le \theta < 4\pi$



Identify and graph the polar equation.

36) $r = 4 \sin(2\theta)$ 36)



Write the complex number in polar form. Express the argument in degrees, rounded to the nearest tenth, if necessary.

37) $1 + \sqrt{3}i$ 37) _____

Write the expression in the standard form a + bi.

38) $\left[\sqrt{3}\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)\right]^4$ 38) _____

Solve the problem. Leave your answer in polar form.

39) $z = 10(\cos 30° + i \sin 30°)$
$w = 5(\cos 10° + i \sin 10°)$
Find zw. 39) _____

40) $z = 1 - i$
$w = 1 - \sqrt{3}i$
Find $\frac{z}{w}$. 40) _____

Find all the complex roots. Leave your answers in polar form with the argument in degrees.

41) The complex cube roots of $-8i$ 41) _____

Find the vertex, focus, and directrix of the parabola with the given equation.

42) $(y - 4)^2 = 12(x + 2)$ 42) _____

A) vertex: (2, −4)
focus: (5, −4)
directrix: $x = -1$

B) vertex: (−2, 4)
focus: (−5, 4)
directrix: $x = 1$

C) vertex: (4, −2)
focus: (7, −2)
directrix: $x = 1$

D) vertex: (−2, 4)
focus: (1, 4)
directrix: $x = -5$

Find an equation for the ellipse satisfying the stated conditions.

43) Vertices at (−2, 4) and (8, 4) ; focus at (6, 4) 43) _____

A) $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{15} = 1$

B) $\frac{(x+3)^2}{9} + \frac{(y+4)^2}{16} = 1$

C) $\frac{(x-3)^2}{49} - \frac{(y+4)^2}{19} = 1$

D) $\frac{(x-3)^2}{25} + \frac{(y-4)^2}{16} = 1$

Find an equation for the hyperbola satisfying the stated conditions.

44) Vertices at $(\pm 7, 0)$; foci at $(\pm 8, 0)$ 44) _____

A) $\frac{x^2}{15} - \frac{y^2}{49} = 1$

B) $\frac{x^2}{49} - \frac{y^2}{15} = 1$

C) $\frac{x^2}{49} - \frac{y^2}{64} = 1$

D) $\frac{x^2}{64} - \frac{y^2}{49} = 1$

Identify the equation without completing the square.

45) $3y^2 - 3x + 2y = 0$ 45) _____

A) ellipse B) parabola C) hyperbola D) not a conic

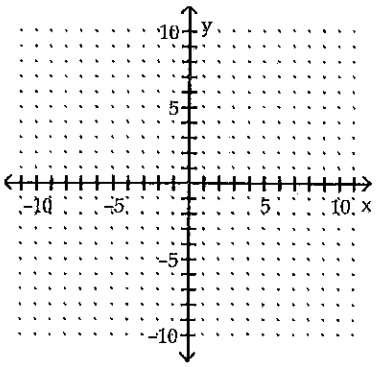
Identify the equation without applying a rotation of axes.

46) $x^2 + 3xy - 3y^2 - 3x + 2y + 3 = 0$ 46) _____

A) ellipse B) parabola C) hyperbola D) not a conic

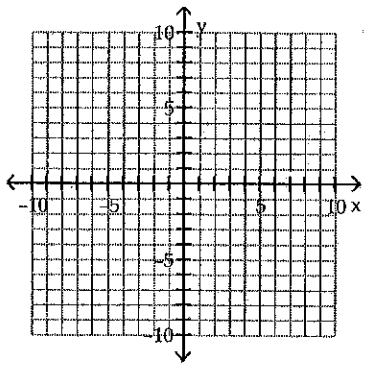
Graph the equation.

47) $9(x + 1)^2 + 16(y - 2)^2 = 144$ 47)



Graph the hyperbola.

48) $\frac{(y + 2)^2}{9} - \frac{(x - 1)^2}{16} = 1$ 48)



Find the value of the determinant.

49) $\begin{vmatrix} -2 & 5 & 4 \\ 3 & -2 & 1 \\ 1 & 6 & -3 \end{vmatrix}$ 49) _____

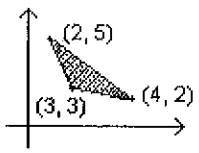
A) 80 B) -12 C) -90 D) 130

Perform the indicated operations and simplify.

50) Let $A = \begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -2 & 8 \\ 1 & 0 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 7 & -9 & 0 \\ 3 & -5 & 1 \\ -1 & 6 & 2 \end{bmatrix}$. Find AB + BC. 50) _____

A) $\begin{bmatrix} 32 & 19 & 40 \\ -15 & 31 & -37 \end{bmatrix}$ B) $\begin{bmatrix} -10 & -19 & 12 \\ -15 & 31 & -25 \end{bmatrix}$ C) $\begin{bmatrix} 32 & 7 & 50 \\ 5 & -23 & -37 \end{bmatrix}$ D) $\begin{bmatrix} 68 & 3 & 31 \\ 8 & -2 & -5 \end{bmatrix}$

Find the maximum or minimum value of the objective function, subject to the constraints graphed in this feasible region.



51) $z = x + 9y + 8$ Find minimum. 51) _____

A) Minimum 39 B) Minimum 30 C) Minimum 55 D) No Minimum

Solve the system of equations.

52) $\begin{cases} 2x - y + 5z = -7 \\ x + y - 2z = -2 \\ x - y + 4z = 8 \end{cases}$ 52) _____

A) inconsistent (no solution)

B) $x = z + 3$
$y = 3z + 1$
$z =$ any real number

C) $x = 3z + 1$
$y = z - 3$
$z =$ any real number

D) $x = -3 - z$
$y = 3z + 1$
$z =$ any real number

Write the partial fraction decomposition of the rational expression.

53) $\dfrac{12x + 3}{(x - 1)(x^2 + x + 1)}$ 53) _____

A) $\dfrac{5}{x - 1} + \dfrac{2x - 5}{x^2 + x + 1}$

B) $\dfrac{-5}{x - 1} + \dfrac{5x + 2}{x^2 + x + 1}$

C) $\dfrac{5}{x - 1} + \dfrac{-5}{x + 1} + \dfrac{2}{x - 1}$

D) $\dfrac{5}{x - 1} + \dfrac{-5x + 2}{x^2 + x + 1}$

Solve the system of equations.

54) $\begin{cases} x + y + z = 7 \\ x - y + 2z = 7 \\ 5x + y + z = 11 \end{cases}$ 54) _____

Solve the problem using matrices.

55) Find real numbers a, b, and c such that the graph of the function $y = ax^2 + bx + c$ contains the points (–2, –4), (1, –1), and (3, –19). 55) _____

56) Melody has $45,000 to invest and wishes to receive an annual income of $4290 from this money. She has chosen investments that pay 5%, 8%, and 12% simple interest. Melody wants to have the amount invested at 12% to be double the amount invested at 8%. How much should she invest at each rate? 56) _____

Use Cramer's rule to solve the linear system.

57) $\begin{cases} -2x + 3y = -3 \\ -2x + 5y = -1 \end{cases}$ 57) _____

Find the inverse of the matrix.

58) $\begin{bmatrix} -4 & 0 \\ -5 & 4 \end{bmatrix}$ 58) _____

Use a graphing utility to find the inverse of the matrix, if it exists. Round answers to two decimal places, if necessary.

59) $\begin{bmatrix} -16 & 3 & 28 \\ 5 & -14 & 15 \\ 34 & 25 & 2 \end{bmatrix}$ 59) _____

Solve the system of equations using substitution.

60) $\begin{cases} y = x^2 - 8x + 16 \\ x + y = 4 \end{cases}$ 60) _____

Solve the system of equations using elimination.

61) $\begin{cases} 3x^2 + 2y^2 = 89 \\ x^2 - 2y^2 = -21 \end{cases}$ 61) _____

Solve the problem.

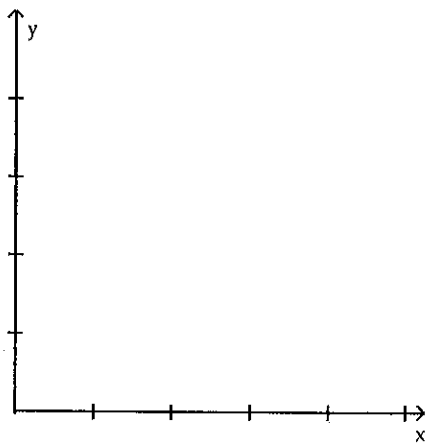
62) The diagonal of the floor of a rectangular office cubicle is 2 ft longer than the length of the cubicle and 5 ft longer than twice the width. Find the dimensions of the cubicle. Round to the nearest tenth, if necessary. 62) _____

Set up the linear programming problem.

63) The Jillson's have up to $75,000 to invest. They decide that they want to have at least $40,000 invested in stable bonds yielding 6% and that no more than $20,000 should be invested in more volatile bonds yielding 12%. 63)

(a) Using x to denote the amount of money invested in the stable bonds and y the amount invested in the more volatile bonds, write a system of linear inequalities that describe the possible amounts of each investment.

(b) Graph the system and label the corner points.



The sequence is defined recursively. Write the first four terms.

64) $a_1 = 2$, $a_2 = 5$ and $a_n = a_{n-2} - 3a_{n-1}$ for $n \geq 3$ 64) _____

A) 2, 5, -1, -16 B) 2, 5, 1, 2 C) 2, 5, -13, 44 D) 2, 5, 17, -46

Express the sum using summation notation with a lower limit of summation not necessarily 1 and with k for the index of summation.

65) $7 + 10 + 13 + 16 + ... + 31$ 65) _____

A) $\sum_{k=1}^{10} 1 + 3k$ B) $\sum_{k=0}^{24} 1 + 3k$ C) $\sum_{k=2}^{10} 1 + 3k$ D) $\sum_{k=2}^{24} 1 + 3k$

Determine whether the given sequence is arithmetic, geometric, or neither. If arithmetic, find the common difference. If geometric, find the common ratio.

66) $\{3n - 4\}$ 66) _____

A) Neither B) Arithmetic, d = -4

C) Geometric, r = 3 D) Arithmetic, d = 3

67) $\{3n^2\}$ 67) _____

A) Arithmetic, d = 3 B) Geometric, $r = \frac{3}{2}$

C) Geometric, r = 3 D) Neither

Use a graphing utility to find the sum of the geometric sequence. Round answer to two decimal places, if necessary.

68) $-3 - 9 - 27 - 81 - 243 - \cdots - 3 \cdot 3^9$ 68) _____

A) -88,535 B) -88,572 C) -88,592 D) -88,570

Find the first term, the common difference, and give a recursive formula for the arithmetic sequence.

69) 8th term is -18; 13th term is -33 69) _____

A) $a_1 = 6,\ d = 3,\ a_n = a_{n-1} + 3$ B) $a_1 = 3,\ d = 3,\ a_n = a_{n-1} + 3$

C) $a_1 = 6,\ d = -3,\ a_n = a_{n-1} - 3$ D) $a_1 = 3,\ d = -3,\ a_n = a_{n-1} - 3$

Find the fifth term and the nth term of the geometric sequence whose initial term, a, and common ratio, r, are given.

70) $a = 5;\ r = -4$ 70) _____

Find the nth term and the indicated term of the arithmetic sequence whose initial term, a, and common difference, d, are given.

71) $a = 7;\ d = 10$ 71) _____
$a_n = ?;\ a_{14} = ?$

Express the sum using summation notation.

72) $\frac{1}{3} + \frac{1}{2} + \frac{3}{5} + \cdots + \frac{15}{17}$ 72) _____

Solve the problem.

73) Jack decided to put $600 into an IRA account every 3 months at a rate of 6% compounded quarterly. Find a recursive formula that represents his balance at the end of each quarter. How long will it be before the value of the account is $100,000? What will be the balance in 30 years when Jack retires? 73) _____

74) A brick staircase has a total of 16 steps The bottom step requires 118 bricks. Each successive step requires 5 less bricks than the prior one. How many bricks are required to build the staircase? 74) _____

A) 2488 bricks B) 1248 bricks C) 2576 bricks D) 1288 bricks

75) Initially, a pendulum swings through an arc of 3 feet. On each successive swing, the length of the arc is 0.8 of the previous length. After 10 swings, what total length will the pendulum have swung (to the nearest tenth of a foot)? 75) _____

Find the sum of the infinite geometric series.

76) $-20 - 5 - \frac{5}{4} - \cdots$ 76) _____

Express the repeating decimal as a fraction in lowest terms.

77) $0.\overline{77} = \frac{77}{100} + \frac{77}{10{,}000} + \frac{77}{1{,}000{,}000} + \cdots$ 77) _____

Use the Principle of Mathematical Induction to show that the statement is true for all natural numbers n.

78) $2 + 5 + 8 + ... + (3n - 1) = \frac{n}{2}(3n + 1)$ 78) _____

Find the indicated coefficient or term.

79) The coefficient of x in the expansion of $(9x + 6)^3$ 79) _____

Expand the expression using the Binomial Theorem.

80) $(2x - 1)^5$ 80) _____

Let A = {q, s, u, w, y}, B = {q, s, y, z}, and C = {v, w, x, y, z}. Find the indicated set.

81) $(A \cap C) \cup (A \cap B)$ 81) _____

A) {q, s, w, y, y} B) {y} C) {q, s, w, y} D) {q, s, u, w, y, z}

Let U = {q, r, s, t, u, v, w, x, y, z}, A = {q, s, u, w, y}, B = {q, s, y, z}, and C = {v, w, x, y, z}. Find the indicated set.

82) $\overline{(A \cap B)}$ 82) _____

A) {s, u, w}

B) {q, s, t, u, v, w, x, y}

C) {r, t, u, v, w, x, z}

D) {t, v, x}

Solve the problem.

83) How many different 11-letter words (real or imaginary) can be formed from the letters in the word ENGINEERING? 83) _____

A) 277,200 B) 554,400 C) 25,200 D) 39,916,800

84) A 6-sided die is rolled. What is the probability of rolling a number less than 2? 84) _____

A) $\frac{5}{6}$ B) $\frac{1}{6}$ C) $\frac{1}{3}$ D) $\frac{1}{9}$

85) In a survey of 161 vacationers in a popular beach resort town, 77 indicated they would consider buying a home there, 73 would consider buying a beach villa, 67 would consider buying a lot, 30 would consider both a home and a villa, 32 would consider both a home and a lot, 30 would consider both a villa and a lot, and 15 would consider all three. How many vacationers would not consider any of the three? How many would consider only a home? 85) _____